

UNIVERSITÉ DE MONTRÉAL



CENTRE DE RECHERCHE SUR LES TRANSPORTS



NETWORK MODELS FOR SEAT ALLOCATION
ON FLIGHTS

by

Moshe Dror^{1,2}
Pierre Trudeau¹
Shaul P. Ladany²

¹ Centre de recherche sur les transports, Université de Montréal.

² Industrial Engineering and Management, Ben Gurion University of The Negev.

To be published in Transportation Research B.

Cette étude a été publiée grâce à une subvention du fonds F.C.A.R. pour l'aide et le soutien à la recherche.

Université de Montréal
Centre de recherche sur les transports - Publication #425
September 1985 - Revised August 1987

Dépôt légal - 1er trimestre 1988

ABSTRACT

This paper examines the problem of proper (optimal) control over the seat allocation on flights. Given a heterogeneous fleet of aircraft types, multi-leg flights, a number of different passenger categories, and cancellations; an airline's objective is to devise an effective system which aids in setting the seat allocation targets for each category of passengers on each flight. This issue is analyzed by a number of authors in the context of economic, simulation based, probabilistic, and mathematical programming studies. We present an attempt to address this problem from the systems prospective emphasizing characteristics such as: passenger cancellations, multi-leg flights, and rolling tactical planning time horizon. Starting from a very simple network flow model for a single flight with a number of intermediate stops, a number of progressively complex models are presented. The airline flights and the seat allocation system are represented as a generalized network flow model (with gains/losses on arcs) with the objective of flow maximization (profit maximization). This modeling approach does not claim to replace the seat allocation approaches presented in Alstrup et al., 1985, Mayer, 1976, Richter, 1982, Simpson, 1985a, and Wang, 1983, but rather construct seat allocations utilizing some of those referenced schemes in a parameter setting mode for a large network model. The objective of this paper is not to report on computational experiments, but to present a modeling approach which seems to be promising, if somewhat speculative.

Key words : airline transportation; seat allocation; generalized network.

- - - - -

RÉSUMÉ

Nous étudierons le problème de la répartition des sièges des avions d'une compagnie aérienne parmi les diverses classes de passagers. Adoptant la perspective de la compagnie, et en tenant compte d'une flotte hétérogène, de vols comportant plusieurs segments, des annulations et des passagers qui ne se présentent pas, nous considérons le problème sur un horizon de temps fini mais dynamique. Quelques modèles simples sont d'abord présentés, pour conclure par un modèle plus complexe de maximisation de flot dans un réseau généralisé avec pénalités sur les arcs.

Mots-clés : transport aérien; répartition des sièges; réseau généralisé.

INTRODUCTION

Over the years airline companies have practiced various promotional pricing policies and devised innovative low fares to develop and stimulate consumer demand and to fulfill the needs of new market segments. As Wang, 1983, points out, this price differentiation was designed to set up barriers to prevent the high yield (business) passengers to switch to the lower-priced promotional fares. Given the passenger price differentiation, the airlines in turn had to decide on an appropriate seat allocation on each flight for the subsequent passenger categories. This was specially important since the low yielding fares tended to block seats for the higher yielding business passengers. Additional blocking phenomena occurs in the case of the multi-leg flight reservations where the long haul passengers are often unable to obtain seats because they are blocked by the shorter haul passengers. For large commercial airline efficiently setting and updating seat allocation targets for each passenger category on each flight is an extremely difficult problem. In addition, the airlines experience cancellations and "no-shows" (unused reservations) for almost all of their flights.

In most of the seat allocation studies originated in the airline industry, the issue of cancellations and no-shows is not addressed (Mayer, 1976, Richter, 1982, Simpson, 1985a and 1985b, Titze and Griesshaber, 1983, Wang, 1983). Moreover, those studies and others (Hersh and Ladany, 1978, and Ladany and Bedi, 1977) tend to examine the seat allocation problem from the local prospective of one flight at a time type analysis. Mayer, 1976, examines the simple and well known models for seat allocation on a single flight and performs sensitivity analysis of these simple formulas. In Hersh and Ladany, 1978, and Ladany and Bedi, 1977, the seat allocation problem for a single flight is analysed from the probabilistic perspective and optimal seat allocation is derived incorporating overbooking. This analysis is of limited value because of its computational requirements and operational scope. Richter, 1982, reexamines the seat allocation problem by concentrating on the marginal effect of switching one seat from a high yield passenger category to a low yield one. The results of such analysis are important in setting allocation goals. Titze and Griesshaber, 1983, conduct a simulation study of the booking behaviour for the low-fare/high-fare seat allocation model. Their conclusion is that the simple formulas remain applicable with different booking behaviours. Wang, 1983, describes a computerized system which aids in setting allocation targets for each passenger type for a single and multi-leg flights. Wang's system attempts to maximize the expected route revenue. Again, each flight-route is examined in isolation. Simpson, 1985a, outlines a theoretical approach to the seat allocation problem taking into account "multiple class, multi-stop flight, and connecting flight situations". The issue of cancellations and no-shows is omitted from the analysis as is the possibility of passenger diversion. Soumis, 1979, in his Ph.D. thesis addresses the broader issue of planning the operational activities for a large commercial airline. The approach to seat allocation as expressed in Soumis, et al., 1979, is that of optimization from the passenger prospective versus the more traditional approach of optimization from the company's point of view. In Soumis, et al., 1979, the authors construct a non-linear programming model with an objective

which incorporates the attractiveness of the routes and the estimated loss of revenue due to the expected number of passengers in excess of the aircraft capacity. The solution methodology used is that of mathematical programming.

In this paper we assume a given regular flight schedule for an aircraft fleet, where a single flight is specified by its itinerary. At this point we assume that a specific aircraft-type is assigned to a specific flight. Subsequently, this assumption will be modified as the modeling methodology is developed. A rolling planning time period composed of time segments is selected (for example four or six weeks) and the seat allocation question is answered at the end of each time segment (daily) for all the flights in the planning period. This is consistent with the assumptions found (among others) in Simpson, 1985a, where it is stated that the probability density for the total number of reservations for a specific flight segment is a function of time before flight, actual bookings at that time and of the day of the week. The approach presented here for optimally solving the seat allocation problem is by finding a solution for a maximal weighted flow problem on a network with gains/losses on some arcs. The models we present are quite flexible and easy to adjust in terms of the number of passenger categories, intermediate stops, and flights considered simultaneously (i.e., weekly or monthly flight schedule).

For a good elementary text on network flow type problem formulations and solution techniques the interested reader is referred to the book by Phillips and Garcia-Diaz, 1981, and a complementary reference for the multicommodity network flows is the paper by Kennington, 1978. In general, it is acknowledged that many network flow problems of realistic size can be solved rather fast on today's computers.

The basic terms in this paper are the same as in Soumis et al., 1979. A "flight" is specified by its itinerary (Montreal-Chicago-Los Angeles), its departure and arrival times, and the aircraft-type used. A "flight-leg" is a non-stop component of a flight (Montreal-Chicago). A route is composed of flight-legs not necessarily belonging to only one flight. For instance a route from Montreal to Dallas may include the flight-leg from Chicago to Dallas.

First, we present a basic network model for seat allocation on a single flight with intermediate stops. This basic network model, though by itself trivial, serves as an important component in the subsequent network models which incorporate many such flights and interrelations resulting from alternative flight choices for passengers. In the computation of the parameters for the main network models, the information about the present reservations levels for the different passenger categories is used in collaboration with other (historical/economical) data. The usual network flow parameters are: upper and lower bounds on arc flows (for example aircraft capacity), gain/loss coefficients for arc flows (expected fraction of cancelations and no-shows), sources and sinks, etc. (More about the relation between this network flow parameter terminology and the usual seat allocation terminology will be clarified as the paper progresses.) The final network model is to be resolved (partially or fully) at the end of each time segment producing new optimal seat allocation goals. The lower and upper bounds on flows together with the gain/loss coefficients are to be updated

automatically at each time segment based on the historical data and the present reservations status.

Basically, given a potential passenger who has indicated the destination, fare category and preferred time of departure, the airline (directly or through a travel agent) either accepts a reservation, or offers the passenger flight alternatives. Any subsequent reservation for this passenger has a different likelihood of cancelation/no-show. All those factors should be incorporated in the seat allocation model, together with the average expected contribution to profit for each of the alternatives selected by the passenger. These alternatives could differ in departure time, aircraft type, and whether the flight is direct, with intermediate stops or even with switchovers.

Now we present the appropriate network flow models.

THE BASIC NETWORK MODEL

The single flight with intermediate stops seat allocation problem is modeled as a maximal flow problem on the following network (Figure 1):

The node s is the source-node and the node t is the sink node. They correspond to flight's origin and final destination respectively. The nodes, c'_i and c_i ($i = 1, 2, \dots, k$), represent category i of passengers. More precisely, the arc (c'_i, c_i) represents category i of passengers. The nodes d_i ($i = 1, 2, \dots, l$) represent the different intermediate stops for the flight. The arc (t', t) represents the aircraft used for that flight. The upper bound (capacity) on the arc (t', t) represents the maximal passenger capacity for this aircraft. The nodes c_i , ($i = 1, 2, \dots, k$) are connected with the nodes d_j , ($j = 1, 2, \dots, l$) according to the destination of the different passenger categories.

The arcs (s, c'_i) have no flow restrictions (lower-upper bound), no cost/profit weight on the flow and no gain/loss flow coefficient. Clearly, originally there is no restriction on the number of passengers who might belong to a certain category (infinite population assumption). The arcs (c'_i, c_i) have only lower and upper bounds on their flow reflecting for that category of passengers their present and maximal (expected or restricted) reservation levels respectively. Arcs (c_i, d_j) are the only arcs with gain/loss flow coefficients (which correspond to 1 minus the expected fraction of cancelations and no-shows) and a positive (might also be negative) weight representing the average expected profit for the corresponding passenger category. Those arcs do not have upper bounds on their flow, only lower (zero) bound which is common to all arcs in this and subsequent models. The arcs (d_i, t') have upper and lower bounds on their

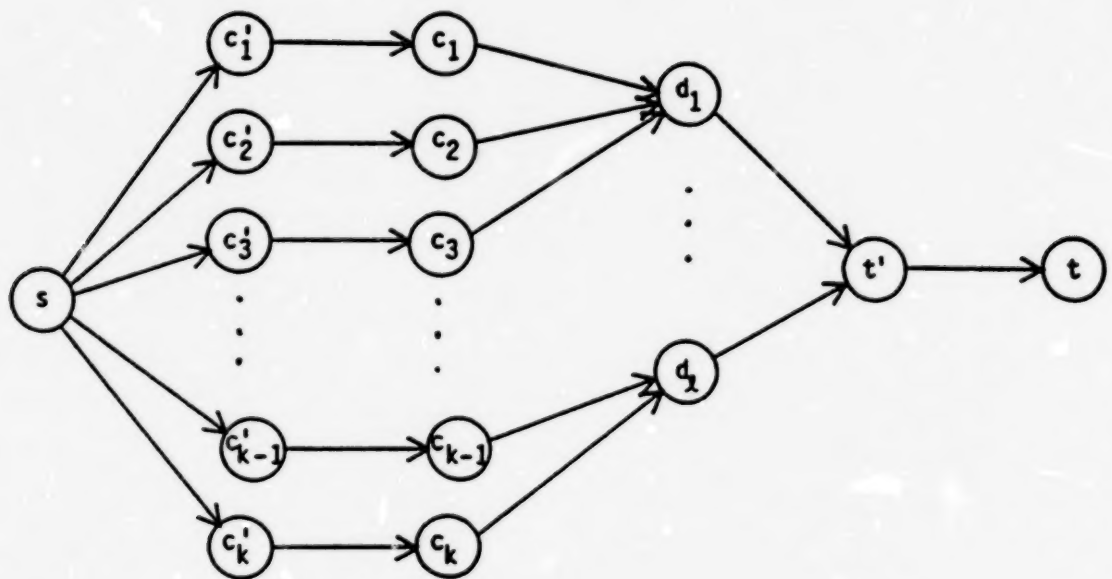


Figure 1 : Single flight network flow model

flow. Those bounds represent allocation limits relative to a specific flight destination. The arc (t', t) has an upper bound on the flow identical with the aircraft's capacity. By solving the maximal weight flow problem on this directed network we obtain the seat allocation on this flight which maximizes the average expected profit.

The solution to this problem on the given network is trivial; Maximize the flow through the path from node s to node t with the maximal profit weight. Once this path is saturated go to the next profitable path and again maximize the weighted flow, etc. The arc weight (profit) exists only on arcs (c_i, d_j) and the paths from node s to node t through these arcs are uniquely determined.

The profit weight on an arc type (c_i, d_j) represents the average expected profit contribution of a passenger from category c_i flying to destination d_j . The gain/loss coefficient on such arc represents 1 minus the expected fraction of cancelations and no-shows for the particular category of passengers. This expected fraction of cancelations and no-shows will vary with time and reservation status before the flight as noted already in the literature.

Illustration

Assume an aircraft (Boeing 767) of 179 passenger capacity and three basic passenger categories: 1st class, 2nd class, and 3rd class. The seats on this aircraft are (usually) partitioned into three sections: 10 seats for the 1st class, 50 seats for the 2nd class, and 119 seats for the 3rd class. Assume a flight with one intermediate stop and no passenger boarding at that stop. In this case we have 6 passenger categories denoted $c_{11}, c_{12}, c_{21}, c_{22}, c_{31}$, and c_{32} indicating the class and the destination. Assume that the average expected profit from passenger flying 1st class to the final destination is 4 units (a simplified measure), for passenger in c_{11} category it is 3 units, for passenger in c_{22} category is 2 units, 1.5 units for passenger in c_{21} , 1.5 units for passenger in c_{32} , and 1 unit of profit from passenger flying to the intermediate stop. In addition, assume the same fraction of cancelations and no-shows for all the passenger classes at 0.10. The gain/loss coefficients (1 minus the cancelations and no-shows fraction) on arcs (c_i, d_j) and the weights on those arcs are now known. The upper bound on the flow through the arc (t', t) is 179 (the capacity of the aircraft). The lower bounds on arcs (s, c'_i) are given by the present reservation levels. The upper bounds are either fixed by specific managerial policy (such as 10 seats, 50 seats and 119 seats for classes 1, 2, and 3 respectively) or calculated accounting for the expected reservation levels given present status. The basic seat partition might fix some upper (and lower) bound parameters if upgrading (i.e., assigning passengers from the 2nd class to 1st class seats) (or downgrading) of passengers is not permitted. The solution in this case is obtained in stright forward fasion by allocating seats according to their expected contribution to profit. The problem gets more complicated when the passenger demand for the flight exceeds the flight capacity and there exists a need to offer passengers alternative flight selections. The cancelation fractions need not be uniform (for business passengers the cancelation and no-show rate might reach 0.40 on some flights) and the average expected profit for passenger category might be

modified accounting for boarding passengers in the intermediate stop. When incorporating more realistic problem parameters, the corresponding network is not any more simple and neither is the solution scheme. These complications are incorporated in the next model.

Note: The above basic network model does not accomplish, so far, anything new. The arc parameters such as bounds on flow, average expected profit margin for passenger category, and gain/loss coefficients, are computed separately. The optimal seat allocation for a single multi-leg flight can be easily obtained through other known techniques. The contribution and novelty of this basic network model emerges in the larger interconnected network models which incorporates many such basic network units and many time periods. This will hopefully become more transparent in the next section.

THE FIRST NETWORK MODEL

In this first network model presented below we interconnect many flights (basic network models) into one single network. This network's time span is the entire (short term) planning period (say four or six weeks). The seat allocation targets are obtained by solving the weighted maximal flow problem on the network. This solution procedure is repeated at the end of each time segment (i.e., daily) with updated network parameters. Passengers in all categories will be offered (directed to) a choice in reservation selection according to the solution obtained in the last run. The figurative network representation of the first network model is illustrated in Figure 2 for one flight over five time segments. The notation (specifically the indices) is slightly different than the one for the basic model in order to incorporate the additional complexities.

The node d_a in Figure 2 is an artificial node representing all the future flights beyond the planning period. Given that the sum of the upper bounds on the flow for some of the flights might exceed the flight capacity of the aircraft (i.e., the demand for a flight exceeds its capacity), the optimal allocation of seats (and diversion of passengers) for the different passenger categories is no longer a trivial problem solved by inspection. The model of course is of no use if all the flights have a low load (demand). Clearly, the model can be also used for re-figuration studies to estimate the revenue impact of additional seats on various routes.

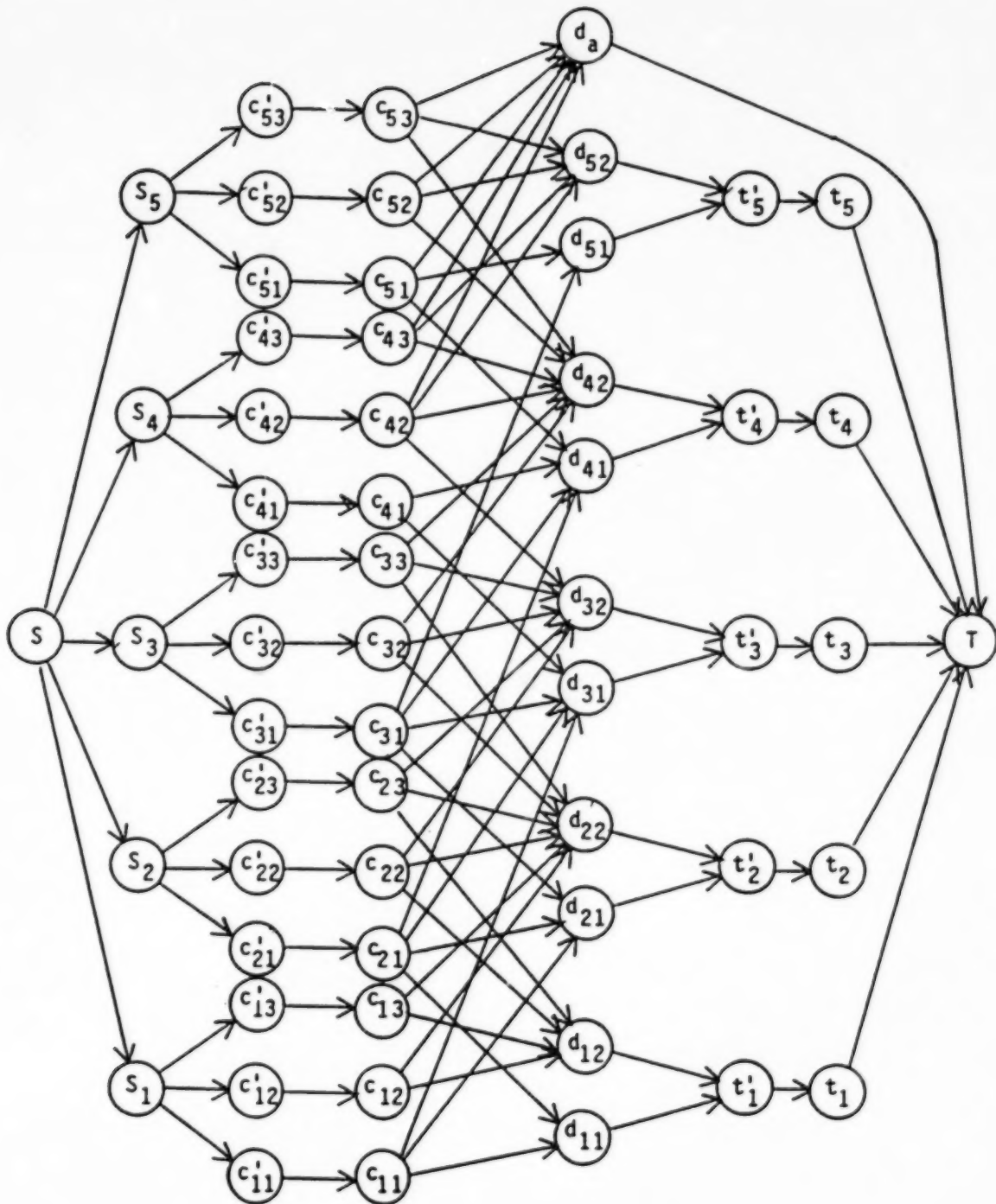


Figure 2 : Five flight network : three customer categories with two identical destinations for each flight

Next, we present the corresponding mathematical formulation for the first network model and explain the notation.

The Mathematical Formulation of the First Network Model

The objective function

$$\text{Maximize } z = \sum_{i,j,l,p} p(c_{ij}, d_{lp}) r_{ij}^{lp} f(c_{ij}, d_{lp})$$

where

$p(c_{ij}, d_{lp})$ is the average expected profit contribution to flight l of one passenger from category j who originally selected flight i and whose destination is d_p .

r_{ij}^{lp} is the fraction of passengers with destination d_p expected not to cancel if in category c_{ij} and scheduled for flight l . One may expect that $r_{ij}^{lp} \geq r_{ij}^{lp}$ for any $l \neq i$.

$f(c_{ij}, d_{lp})$ is the flow - (the number of reservations) - that should be allowed on the arc from c_{ij} to d_{lp} .

Constraints :

$l(x, y) \leq f(x, y) \leq u(x, y)$ for all arcs (x, y) in the network as indicated. $u(x, y)$ is the upper bound on the flow from node x to node y and $l(x, y)$ is the lower bound on the flow from node x to node y .

$$f(s, s_i) - \sum_j f(s_i, c'_{ij}) = 0, \quad \text{for all } i$$

$$f(s_i, c'_{ij}) - f(c'_{ij}, c_{ij}) = 0, \quad \text{for all } i, j$$

$$\sum_{i,j \in I_p} r_{ij}^{lp} f(c_{ij}, d_{lp}) - f(d_{lp}, t'_l) = 0, \quad \text{for all } l, p$$

$$I_{lp} = \{i, j \text{ such that the arc } (c_{ij}, d_{lp}) \text{ exists}\}$$

$$f(c'_{ij}, c_{ij}) - \sum_{l,p \in J_{ij}} f(c_{ij}, d_{lp}) = 0, \quad \text{for all } i, j$$

$$J_{ij} = \{l, p \text{ such that the arc } (c_{ij}, d_{lp}) \text{ exists}\}$$

At this point we note that this network optimization model is not restricted to identical planes or identical flights from an origin to a final destination. The precise indication as to the type of aircraft on a given flight is via the upper bound on the flight's capacity, i.e., the upper bound on the flow through the arc (t'_i, t_i) . Different flights with even one overlapping intermediate stop (which could be a final destination for one flight) can also be incorporated in this flow model. Needless to say that this model can also be implemented for a bus company with long routes and intermediate stops.

In order to clarify the above network flow model, we examine the interrelation between the lower and upper bounds on the different arcs in the model.

The lower bound on the flow through the arc type (c'_{ij}, c_{ij}) indicates the present reservation level for the passenger-category j whose initial choice was the flight i . The upper bound on the flow through that arc indicates either the expected demand for that passenger category and flight given the present reservation status, or some strategic/managerial restriction on reservation for a certain (low yield) category of passengers. The lower bound on the flow through the arc type (c_{ij}, d_{lp}) represents the present reservation status on flight l for passengers of the j^{th} category whose initial choice was flight i and whose destination is d_p . These lower bounds multiplied by the noncancelation fraction and summed for a specific l, p over all ij 's in the connecting arcs are the lower bounds on the arc (d_{lp}, t'_p) . The upper bound on the flow through arc (d_{lp}, t'_p) can represent the expected demand for the destination d_p on flight l . The lower bound on flow through the arc (t'_p, t_p) is the sum of the lower bounds on the arcs (d_{lp}, t'_p) and the upper bound is the actual number of seats on the flight p .

This network flow model does not explicitly permit the possibility of passengers boarding a flight at an intermediate stop. This eventuality can be incorporated implicitly into this model by properly adjusting the average expected profit margin for the passenger flying only from the origin to an intermediate stop by the average expected additional profit for that plane seat in a flight-leg from that intermediate stop to the final destination.

One major drawback of this first network model is its inability to incorporate the case in which passengers are allowed to switch planes at an intermediate stop in order to arrive at their final destination. It is an important aspect to be considered when setting allocation goals for seats on flights, since approximately 50% of the passengers switch flights at an intermediate stops.

In order to include the possibility for passengers to switch planes at an intermediate stops, a more complex version of the network flow model is required. This quite complex network flow model is the topic of the next section.

THE SECOND (EXPANDED) NETWORK MODEL

The second (expanded) network model is a (structured) version of a maximal flow multicommodity network flow problem with gains/losses on arcs. We illustrate the expanded network flow model which incorporates the possibility of switch-over passengers for the simple case of two planes with one intermediate stop each and six basic categories of passengers according to the origin destination pair. The first illustration is conceptual and presented in Figure 3 and the second illustration is in the appropriate network configuration (Figure 4) with passenger categories according to class. The special structure of this multicommodity network flow problem is represented by requiring identical flows on some specific pairs of nonadjacent arcs.

It is clearly very easy to incorporate directly passengers boarding the planes at the intermediate stops.

The Mathematical Formulation of the Second (Expanded) Network Model

Notation :

s : source node

t : sink node

(c'_{ij}, c_{ij}) : j^{th} category of passengers on flight i

(c^*_{ij}, c_{ij}) : j^{th} category of passengers on flight i but boarding at an intermediate stop.

$\bar{d}_{ij}$: denotes a node which corresponds to the destination d_j which is final for some passengers of flight i departing at an intermediate stop. Note that destination d_j may be common to many flights, for instance flights a and b will have $\bar{d}_{aj}, \bar{d}_{bj}$.

d_{ij}^k : denotes a node which corresponds to destination d_j in flight i , and is related to passengers entering the flight after the k^{th} flight leg ($k \geq 0$).

$\tilde{d}_{ij}^k$: denotes a node which corresponds to a switch-over in flight i , and is related to passengers entering the flight i after the k^{th} flight leg and traveling to destination d_j .

$l(x, y)$: lower bound constraint on the flow from node x to node y

$u(x, y)$: upper bound constraint on the flow from node x to node y

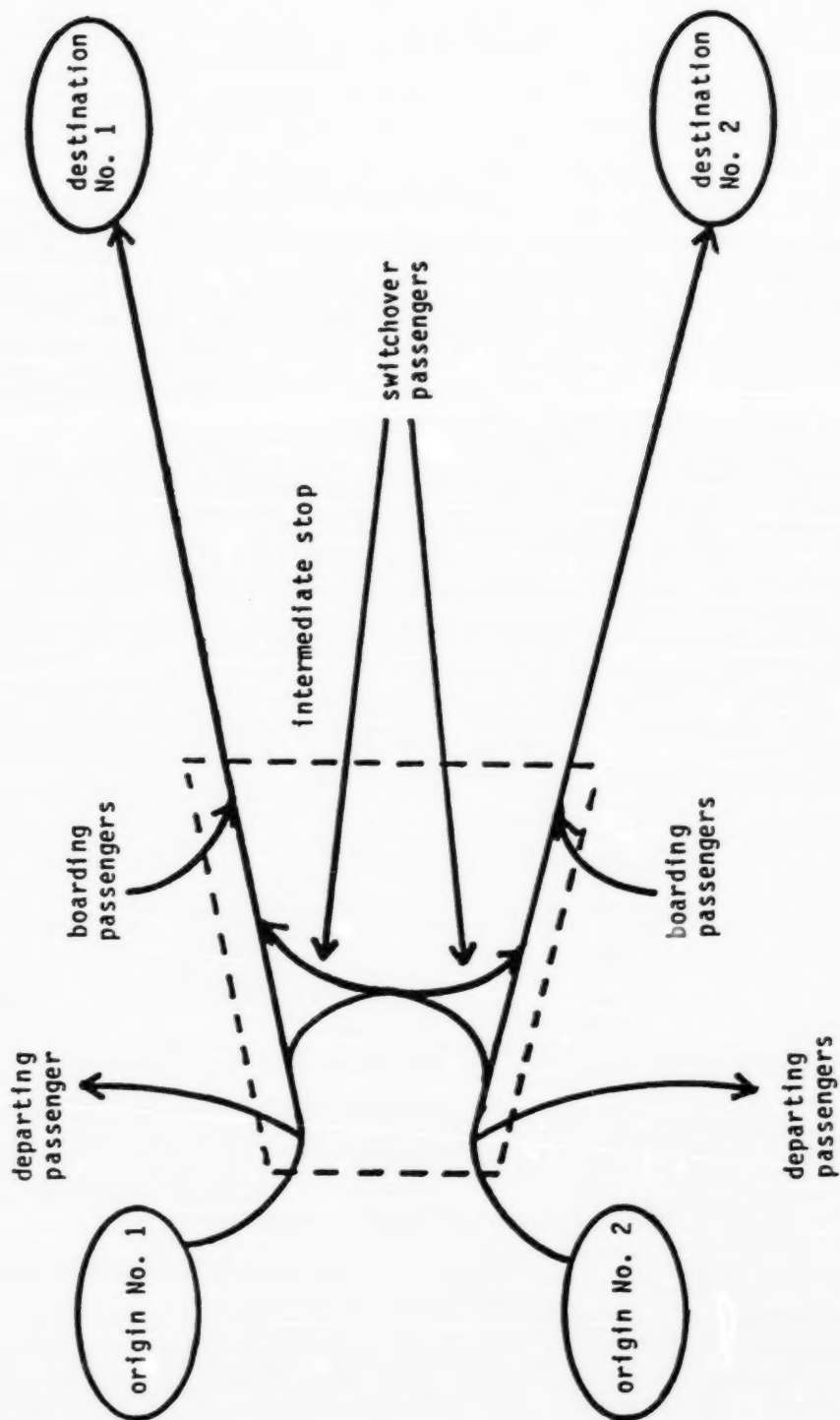


Figure 3 : A schematic presentation of two flights with one switch-over intermediate stop

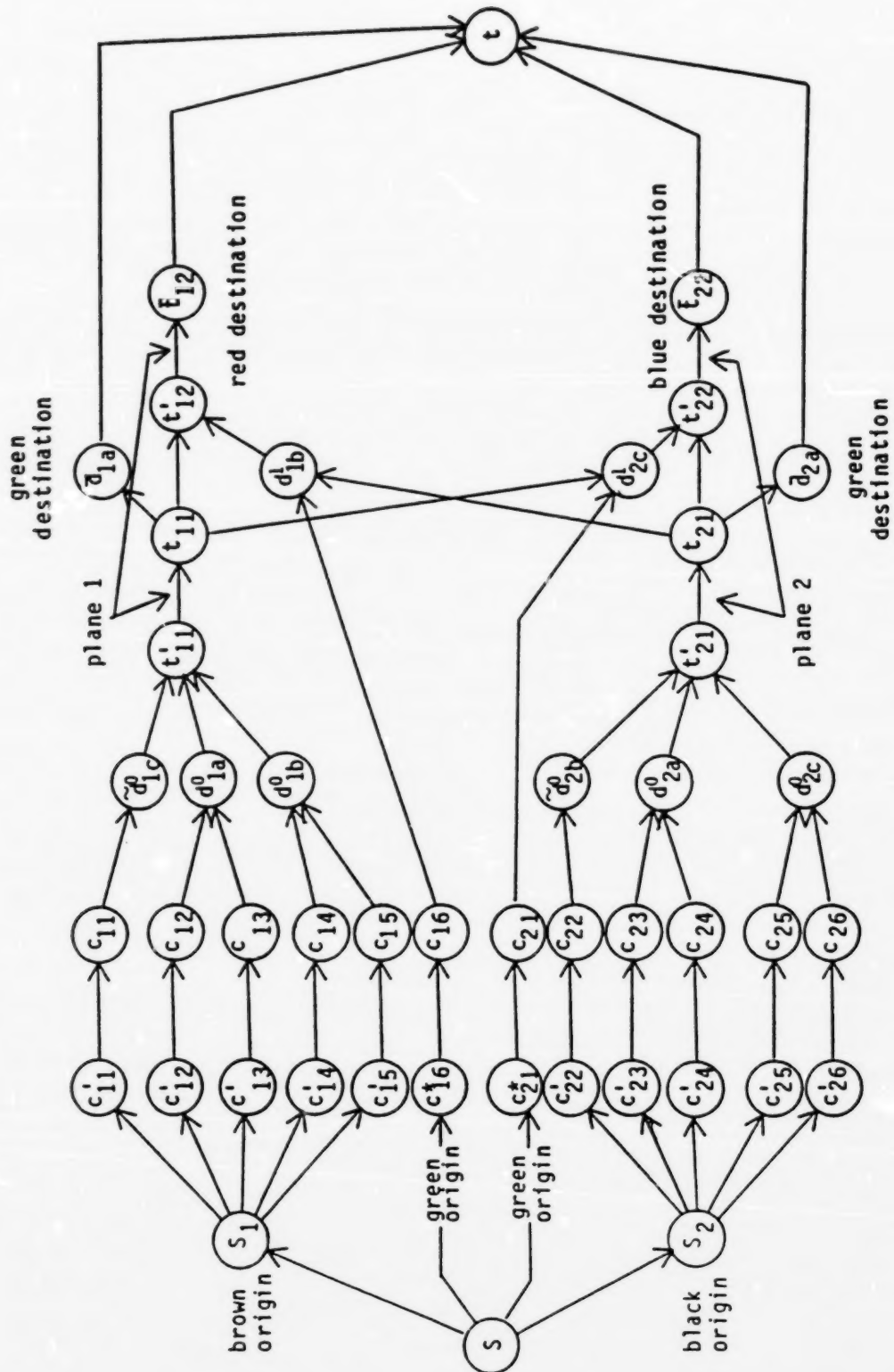


Figure 4 : Two flights with one intermediate switch-over stop

r_{ij}^{klp} : fraction of passengers with destination d_p expected not to cancel if in category c_{ij} and assigned flight l , boarding after the k^{th} flight leg and making no switch-over thereafter.

$\tilde{r}_{ij}^{klp}$: fraction of passengers with destination d_p expected not to cancel if in category c_{ij} and assigned flight l , boarding after the k^{th} flight leg and switching over later to eventually reach destination d_p .

(t'_{ij}, t_{ij}) : denotes the j^{th} flight-leg of flight i .

$\bar{l}_{ij}$: denotes the final destination of flight i occurring on its j^{th} stop.

$I_{ij} = \{l, k \text{ such that the arc } (c_{ij}, d_{lp}^k) \text{ exists, where } d_p \text{ is destination of } c_{ij}\}.$

$\tilde{I}_{ij} = \{l, k \text{ such that the arc } (c_{ij}, \tilde{d}_{lp}^k) \text{ exists, where } d_p \text{ is the destination of } c_{ij}\}.$

$J_{klp} = \{i, j \text{ such that the arc } (c_{ij}, d_{lp}^k) \text{ exists, where } d_p \text{ is the destination of } c_{ij}\}.$

$\tilde{J}_{klp} = \{i, j \text{ such that the arc } (c_{ij}, \tilde{d}_{lp}^k) \text{ exists, where } d_p \text{ is the destination of } c_{ij}\}.$

$K_{klp} = \{m, n \text{ such that the arc } (t_{mn}, d_{lp}^k) \text{ exists}\}$

$\tilde{K}_{klp} = \{m, n \text{ such that the arc } (t_{mn}, \tilde{d}_{lp}^k) \text{ exists}\}$

The notation (above) is aimed at representing in a detailed manner all the intricacies of this complex version of the seat allocation problem. We did not aim to present the most concise (in terms of variables) representation of the problem, but rather attempted to be descriptive.

The objective function :

$$\begin{aligned} \text{Maximize } z = & \sum_{i,j,k,l,p} p(c_{ij}, \tilde{d}_{lp}^k) \tilde{r}_{ij}^{klp} f(c_{ij}, \tilde{d}_{lp}^k) \\ & + \sum_{i,j,k,l,p} p(c_{ij}, d_{lp}^k) r_{ij}^{klp} f(c_{ij}, d_{lp}^k) \end{aligned}$$

Constraints :

$$l(x, y) \leq f(x, y) \leq u(x, y) \quad \text{for all arcs } (x, y)$$

For the flow constraints below, the symbol in the left margin indicates the node for which the conservation of flow is about.

$$s_i : f(s, s_i) = \sum_j f(s_i, c_{ij}^l) \quad \text{for all } i$$

$$c_{ij}^* : f(s, c_{ij}^*) = f(c_{ij}^*, c_{ij}) \quad \text{for all such } (i, j)$$

$$c'_{ij} : f(s_i, c'_{ij}) = f(c'_{ij}, c_{ij}) \quad \text{for all such } (i,j)$$

$$c_{ij} : f(c'_{ij}, c_{ij}) = \sum_{l,k \in I_{ij}} f(c_{ij}, d_{lp}^k) + \sum_{l,k \in \widetilde{I}_{ij}} f(c_{ij}, \widetilde{d}_{lp}^k)$$

for all such (i,j) where d_p is the destination of c_{ij}

$$c_{ij} : f(c_{ij}^*, c_{ij}) = \sum_{l,k \in I_{ij}} f(c_{ij}, d_{lp}^k) + \sum_{l,k \in \widetilde{I}_{ij}} f(c_{ij}, \widetilde{d}_{lp}^k)$$

for all such (i,j) where d_p is the destination of c_{ij}

$$\widetilde{d}_{lp}^k : \sum_{i,j \in \widetilde{J}_{klp}} f(c_{ij}, \widetilde{d}_{lp}^k) r_{ij}^{klp} + \sum_{m,n \in \widetilde{K}_{klp}} f(t_{mn}, \widetilde{d}_{lp}^k) = f(\widetilde{d}_{lp}^k, t'_{l,k+1})$$

for all such (k,l,p) where d_p is the destination of c_{ij}

$$d_{lp}^k : \sum_{i,j \in J_{klp}} f(c_{ij}, d_{lp}^k) r_{ij}^{klp} + \sum_{m,n \in K_{klp}} f(t_{mn}, d_{lp}^k) = f(d_{lp}^k, t'_{l,k+1})$$

for all such (k,l,p) where d_p is the destination of c_{ij}

$$t'_{l1} : \sum_{p \in D_l} f(\widetilde{d}_{lp}^0, t'_{l1}) + \sum_{p \in D_l} f(d_{lp}^0, t'_{l1}) = f(t'_{l1}, t_{l1}) \quad \text{for all } l \text{ where } D_l = \{p; d_p \text{ is a destination of flight } l\}$$

$$t'_{lk} (k \geq 2) : f(t_{l,k-1}, t'_{lk}) + \sum_p f(d_{lp}^{k-1}, t'_{lk}) = f(t'_{lk}, t_{lk}) + \sum_p f(\widetilde{d}_{lp}^{k-1}, t'_{lk}) \quad \text{for all } l, k (k \geq 2)$$

$$t_{lk} : f(t'_{lk}, t_{lk}) = f(t_{lk}, \bar{d}_{ln}) + f(t_{lk}, t'_{l,k+1}) + \sum_{ijr} f(t_{lk}, d_{ij}^r) + \sum_{ijr} f(t_{lk}, \widetilde{d}_{ij}^r)$$

for all l, k where d_n is the k^{th} destination on flight l .

$$\bar{d}_{ln} : f(t_{lp}, \bar{d}_{ln}) = f(\bar{d}_{ln}, t) \quad \text{for all } l, n \text{ where } d_n \text{ is the } p^{th} \text{ destination on flight } l.$$

$$\bar{t}_{lp} : \sum_l f(t'_{lp}, \bar{t}_{lp}) = \sum_l f(\bar{t}_{lp}, t) \quad \text{for all } l \text{ where } p \text{ is the number of destinations on flight } l.$$

The next two equations express equal flow requirements between (specific) non-adjacent arcs. The equations are referred to as 'switch-over' and 'out' respectively.

$$\text{switch-over} : f(\widetilde{d}_{lp}^k, t'_{l,k+1}) = \sum_{n>k} \sum_{r,i} f(t_{ln}, \widetilde{d}_{ip}^r) + \sum_{n>k} \sum_{r,i} f(t_{ln}, d_{ip}^r) \quad \text{for all such } k, l, p$$

$$\text{out} : f(t_{lp}, \bar{d}_{ln}) = \sum_{1 \leq i \leq p} f(d_{ln}^{i-1}, t'_{li}) \quad \text{for all } l, p \text{ where } d_n \text{ is the } p^{th} \text{ destination on flight } l.$$

We do not claim that the above mathematical model can be easily solved. The model presents a very detailed (though still partial) and complex representation of the routine activities for a commercial airline. In order to solve this type of a model for a large operation, in our estimation, new solution techniques will have to be examined and developed. Most of the necessary methodology exists and would have to be suitably pieced together into appropriate solution procedure.

Figure 5 presents the network flow illustration for the mathematical description of this Second (expanded) Network Flow Model. Three flights are represented (1,2, and 3), originating at three different cities (black, brown, and white), and a total of six destinations (yellow, blue, orange, green, red, and white). To simplify, each category of passengers of a given flight is going to a different destination. Category c_{15} of passengers illustrates the possibility of multi switch-overs (two in this case) starting in flight 1, transferring to flight 2 in the yellow city and finally leaving flight 2 in the white city for flight 3 where it will reach the desired destination, red city.

REMARKS ON NETWORK PARAMETERS AND UPDATING PROCEDURE

The objective for the airline is to construct an operating policy for the allocation of seats to passengers flying full flights and flight-legs such that the expected total contribution to profit for the entire flight schedule over a planning period is maximum. Since the demands and cancelations for the full span and the intermediate span are stochastic variables dependent on the time prior the flight departure and other factors, it becomes necessary to derive the allocation policy as a sequential decision process. At the start of each t^{th} decision period, prior to the departure, a decision must be made about the maximum additional number of reservations to be allowed for the different passenger categories in the different flights and flight-legs. This decision must be based upon :

- (a) The aircrafts' capacities. In this problem the aircraft' capacities for the different flights can be varied in order to examine the relationship between the maximum flow(i.e., maximum profit solution) and the aircraft assignment to the different flights.
- (b) The average expected contribution to profit of a passenger from a given category on a given flight-leg.
- (c) The probabilistic pattern of future demands, cancelations, and no-shows, for each category of passengers and flight-legs.
- (d) The distribution of standbys/waiting list passengers for each flight leg.
- (e) The present status of reservation for each passenger category and flight-leg.
- (f) The penalty cost incurred for passengers with confirmed reservation but no seat available (overbooking cost).

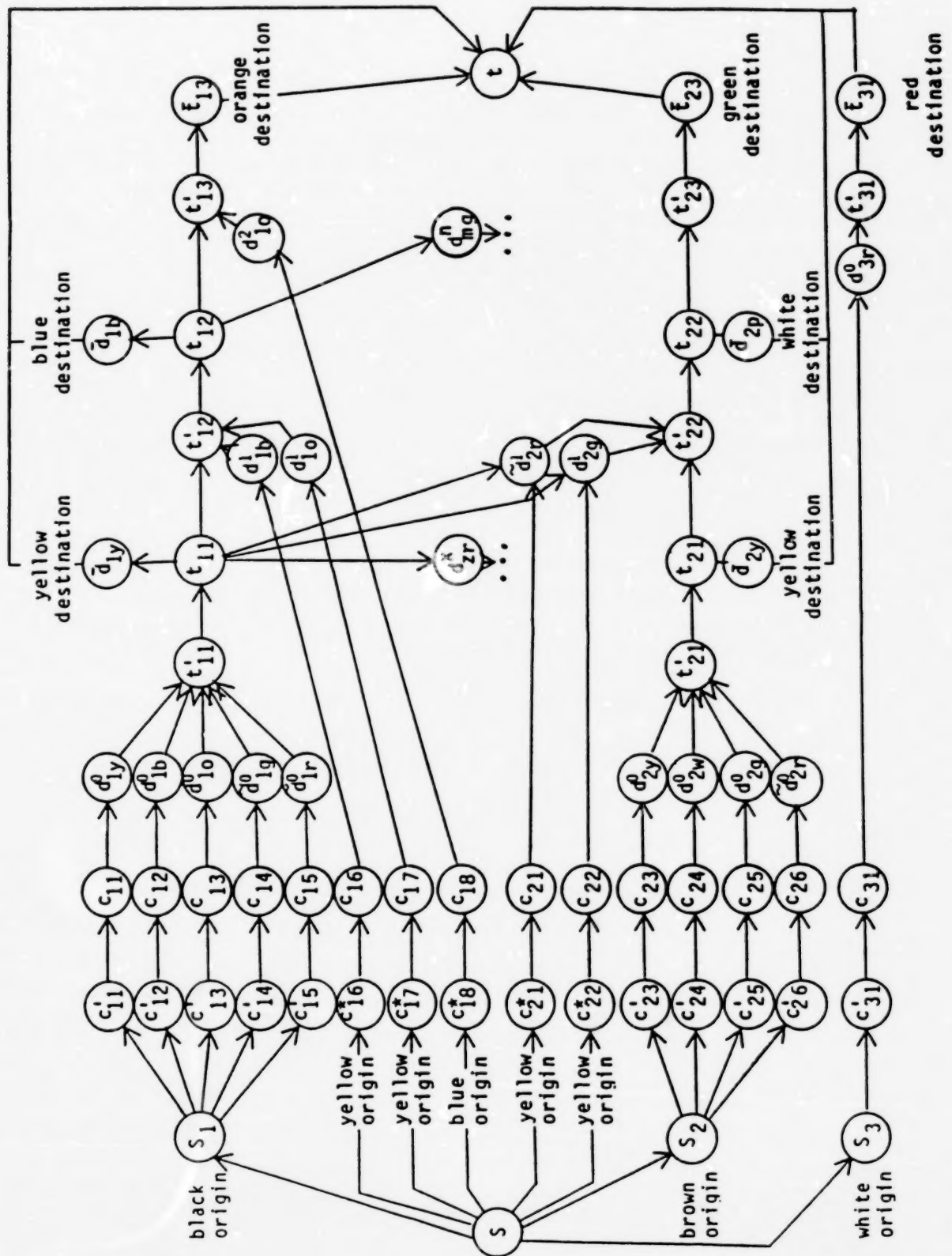


Figure 5 : Three flights with a 2-switch over example

CONCLUSION

In this paper we presented a new modeling approach of the airline operation in respect to allocation of seats on flights to different passenger categories. In contrast to most of the other approaches to this problem, we presented a systems approach which examines the seat allocation for an entire fleet of aircraft types over a planning period. We hope to have demonstrated that this modeling approach is quite flexible in terms of the planning period and the flights as well as a large number of passenger categories.

The model could be implemented in relatively simple cases (a subset of flights) where no switch-over passenger are allowed, in a rather straightforward mode of solving the maximal weight flow problem on a network with gains/losses. When incorporating the switch-over passengers into the model, the corresponding network evolves into a more complex multicommodity network flow problem to which additional constraints are added.

There are two major aspects of the present models which have not been discussed in this paper. (i) The calculation of the parameters for this type of networks, and (ii) The specifics of the computerized procedures (the algorithms) which can solve these network models. Those issues require a separate forum.

REFERENCES

- Alstrup, J., Boas, S., Madsen, O.B.G., and Vidal, R.V.V. (1985) "Booking policy for flights with passenger types", IMSOR, The Technical University of Denmark, Research Report No. 01.
- Hersh, M., and Ladany, S.P. (1978) "Optimal seat allocation for flights with intermediate stops", **Computers and Operations Research**, 5, 31-37.
- Kennington, J. (1978) "A survey of linear cost multicommodity network flows", **Operations Research**, 26, 209-236.
- Ladany, S.P., and Bedi, D.N. (1977) "Dynamic booking rules for flights with intermediate stop", **Omega**, 5, 6, 721-730.
- Mayer, M. (1976) "Seat allocation, or a simple model of seat allocation via sophisticated ones", **Proceedings of the AGIFORS Sixteen Annual Symposium**, Miami, Florida, 103-125, September.
- Phillips, D.T., Garcia-Diaz, A. (1981) *Fundamentals Of Network Analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Richter, H. (1982) "The differential revenue method to determine optimal seat allotments by fare types", **Proceedings of the AGIFORS Twenty Second Annual Symposium**, Longonissi, Greece, 339-353, October.
- Simpson, R.W. (1985a) "Setting optimal booking levels for flight segments with multi-class, multi-market traffic", **Proceedings of the AGIFORS Twenty-Fifth Annual Symposium**, Hikone, Japan, 263-271, October.
- Simpson, R.W. (1985b) "Theoretical concept for capacity/yield management", **Proceedings of the AGIFORS Twenty-Fifth Annual Symposium**, Hikone, Japan, 281-293, October.
- Soumis, F. (1979) "Planification d'une flotte d'avions", Ph.D. Thesis, Departement d'informatique et de recherche operationnelle, Universite de Montreal, Publication # 133. Centre de recherche sur les transports, Universite de Montreal.
- Soumis, F., Ferland, J.A., and Rousseau, J.M. (1979) "MAPUM - a model for assigning passengers to flight schedule", Publication # 142, Centre de recherche sur les transports, Universite de Montreal.
- Titze, B., and Griesshaber, R. (1983) "Realistic passenger booking behaviour and the simple low-fare/high-fare seat allotment model", **Proceedings of the AGIFORS Twenty Third Annual Symposium**, Olive Branch, Mississippi, 197-223, October.
- Wang, K. (1983) "Optimum seat allocation for multi-leg flights with multiple fare types", **Proceedings of the AGIFORS Twenty Third Annual Symposium**, Olive Branch, Mississippi, 225-246, October.